Properties of SASE FEL pulses

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What determines the FEL property

The field of a SASE FEL (by solving Green's function) is

$$E(t,z) = E_0(z)\sum_{j=i}^{N_e}\exp\left[i\omega_0[1+c\frac{\sigma_\delta}{\sigma_z}(t-t_0)](t-t_j)-\frac{(t-t_j-z/v_g)^2}{4\sigma_t^2}(1-\frac{i}{\sqrt{3}})\right],$$

$$\approx E_0(z)\sum_{j=i}^{N_e}\exp\left[i\omega_0(t-t_j)-\frac{(t-t_j-z/v_g)^2}{4\sigma_t^2}\right].$$

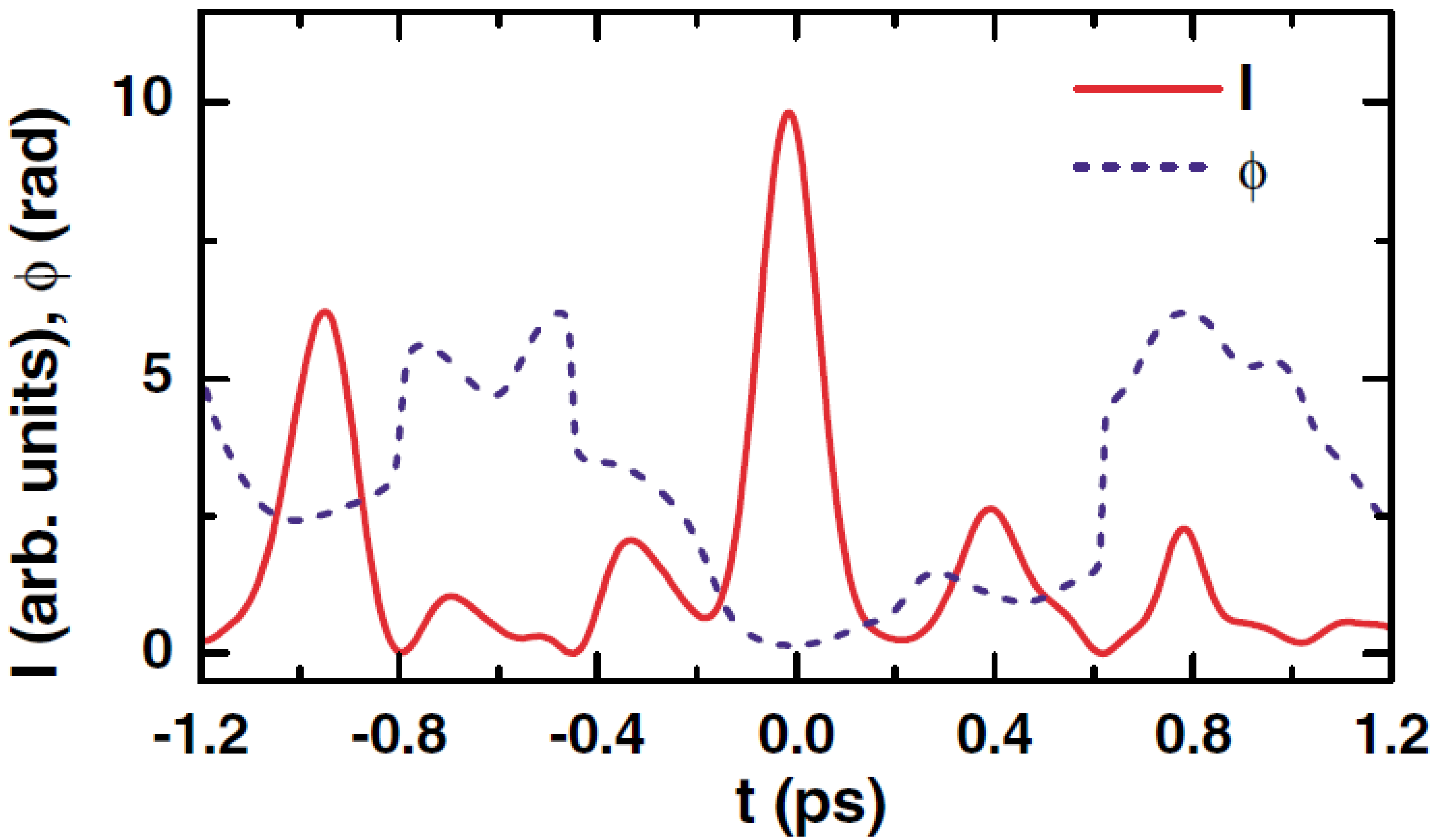
[S. Krinsky and Z. Huang, Phys. Rev. ST Accel. Beams 6, 050702 (2003).]

ω_0	resonant frequency	$\omega_0 = \dfrac{4\pi c\gamma_0^2}{\lambda_u(1+K^2/2)}$
σ_t	coherence length	$\sigma_t = \dfrac{1}{2\omega_0}\sqrt{\dfrac{z}{\rho\lambda_u}} \propto n_e^{-1/4}$
σ_δ/σ_z	electron beam energy chirp	

Summing of random phasors: Chaotic light

Goodman, Statistical Optics, (John Wiley & Sons, New York, 1985), p. 35. S. Krinsky, PRSTAB 6, 050701 (2003).

But the result is remarkable



How is the coherence built

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Resonance Condition

Question: How can an optical wave traveling at the speed of light interact with slower electrons in a fast wave device (e.g., FEL)?

Answer: If the optical wave slips ahead of the electrons exactly one wavelength every wiggler period, the sum of wiggler phase and optical phase is constant, and energy exchange can occur.

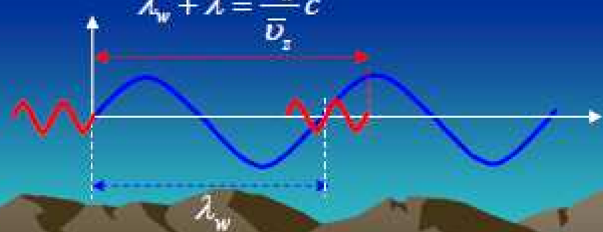
$$\theta = (k_w + k)z - \omega t = const.$$

$$\frac{d\theta}{dz} = k_w + k - \frac{\omega}{\overline{\upsilon}_z} = 0$$

$$k_w + k = \frac{k}{\left[1 - \frac{1+a_w^2}{2\gamma^2}\right]} \approx k + k\left(\frac{1+a_w^2}{2\gamma^2}\right)$$

Resonance wavelength satisfies this condition

$$\lambda_w + \lambda = \frac{\lambda_w}{\overline{\upsilon}_z}c$$



$$k_w = k\left(\frac{1+a_w^2}{2\gamma^2}\right) \rightarrow \boxed{\lambda = \lambda_w\left(\frac{1+a_w^2}{2\gamma^2}\right)}$$

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The FEL parameter

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High-Gain FEL

Dimensionless Pierce parameter as a function of k_w (left) or λ_w (right)

$$\boxed{\rho = \frac{1}{2\gamma}\left(\frac{[JJ]a_w}{\sigma k_w}\right)^{\frac{2}{3}}\left(\frac{I}{I_A}\right)^{\frac{1}{3}}} \qquad \boxed{\rho = \frac{1}{\gamma}\left(\frac{[JJ]a_w\lambda_w}{4\sqrt{2}\pi\sigma}\right)^{\frac{2}{3}}\left(\frac{I}{I_A}\right)^{\frac{1}{3}}}$$

Recall JJ is the difference between J_0 and J_1 Bessel functions of argument ξ

$$[JJ] = J_0(\xi) - J_1(\xi)$$

$$J_0(\xi) \approx 1 - \frac{\xi^2}{4} \qquad J_1(\xi) \approx \frac{\xi}{2}$$

$$[JJ] \approx 1 - \frac{\xi}{2} - \frac{\xi^2}{4}$$

where

$$\xi = \frac{a_w^2}{2\left(1 + a_w^2\right)}$$

High gain FEL is applicable in a long wiggler driven by a high-brightness electron beam (one with high peak current and small emittance). The wiggler length must be significantly longer than the power gain length, given by

Power gain length $\boxed{L_G = \dfrac{\lambda_w}{4\pi\sqrt{3}\rho}}$

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The coherence length rough estimate

$$L_c \approx \frac{L_G}{\lambda_w}\lambda = \frac{\lambda_w}{4\pi\sqrt{3}\rho}\frac{1}{\lambda_w}\lambda = \frac{\lambda}{4\pi\sqrt{3}\rho}$$

$$\sigma_t = \frac{L_c}{c} = \frac{1}{2\omega\sqrt{3}\rho}$$

$$\sigma_\omega = \frac{\sqrt{\pi}}{\sigma_t} = 2\omega\sqrt{3\pi}\rho.$$

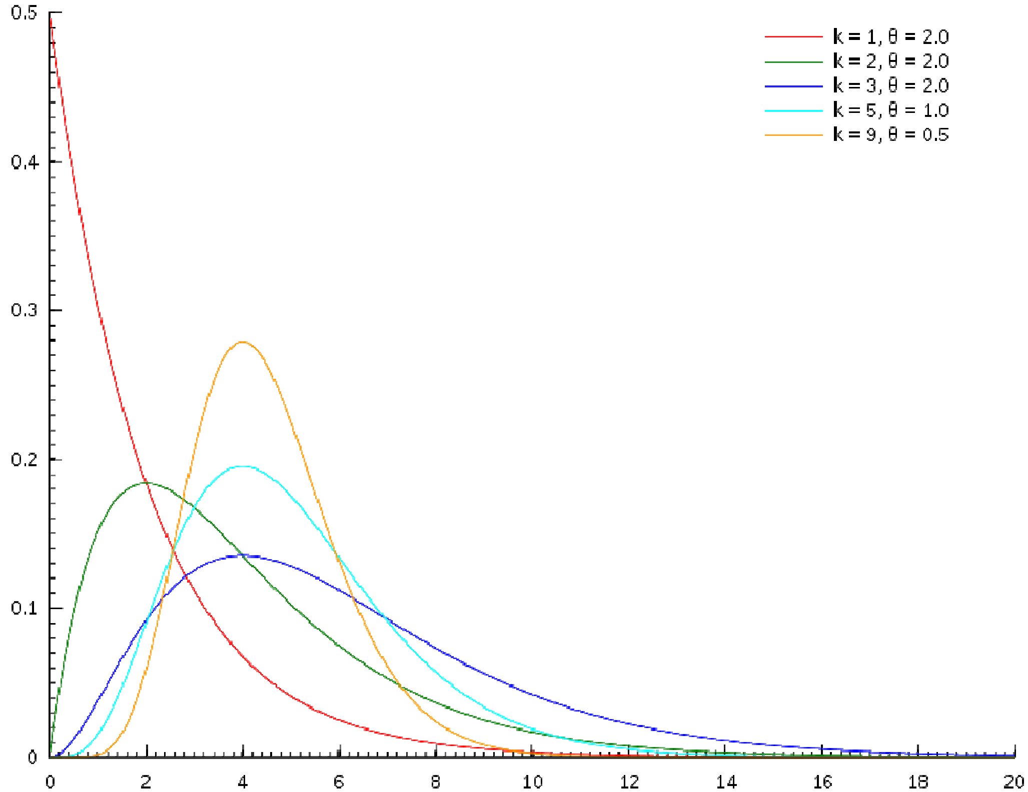
$$\sigma_t = \frac{1}{2\omega_0}\sqrt{\frac{z}{\rho\lambda_u}}$$

$$\sigma_\omega = \omega_r\sqrt{3\sqrt{3}\rho/k_u z}$$

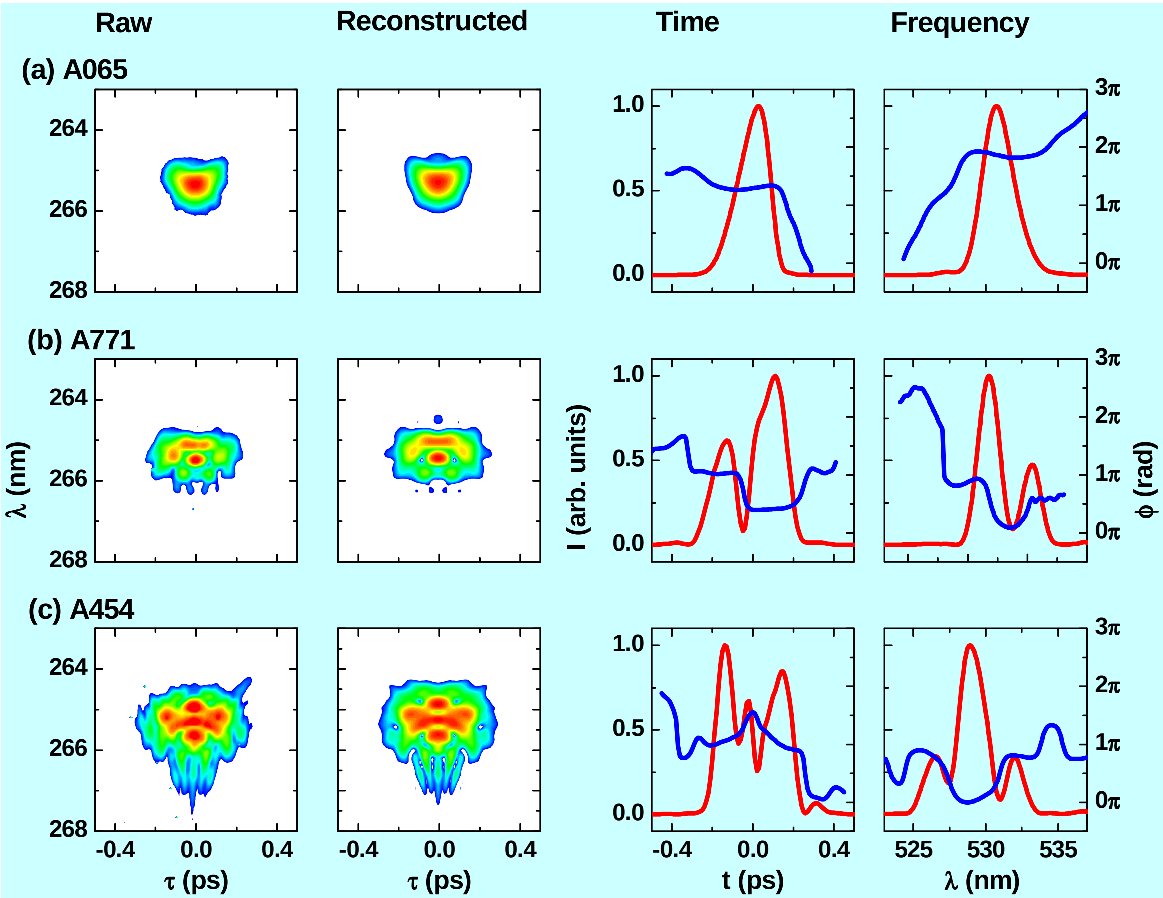
Total energy fluctuation

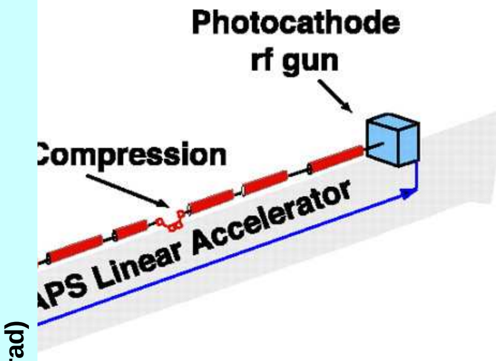
$$M \approx \frac{T_B}{\sigma_t}$$

$$p(E) = \frac{M^M}{\Gamma(M)} \left(\frac{E}{\langle E \rangle} \right)^{M-1} \frac{1}{\langle E \rangle} \exp\left(-M \frac{E}{\langle E \rangle} \right)$$



The FROG experiment at 530 nm





rms coherence length σ_t (fs) [b]	53±16
Time bandwidth product	0.52±0.31
FWHM pulse duration (fs)	197±105
FWHM bandwidth (nm)	2.6±0.8

The single-shot FROG technique

= Frequency Resolved Optical Gating

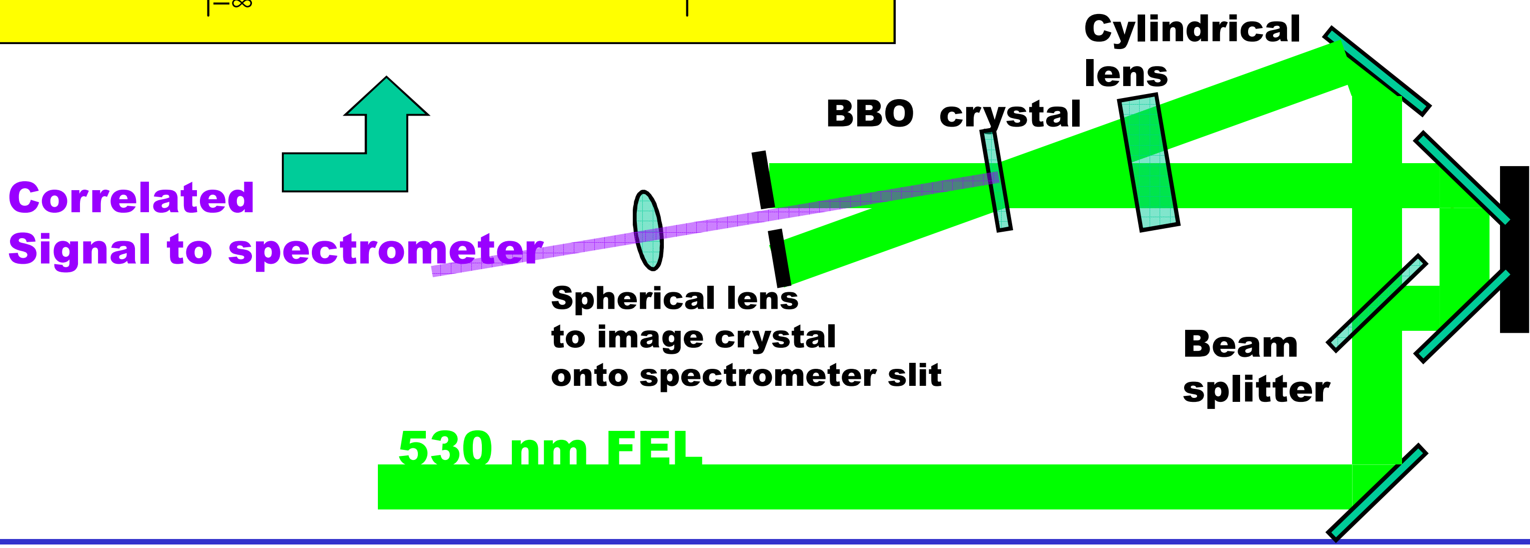
Kane and Trebino, JQE, 29, 571 (1993).
DeLong and Trebino, JOSA B, 11, 2206 (1994).

For the second harmonic FROG

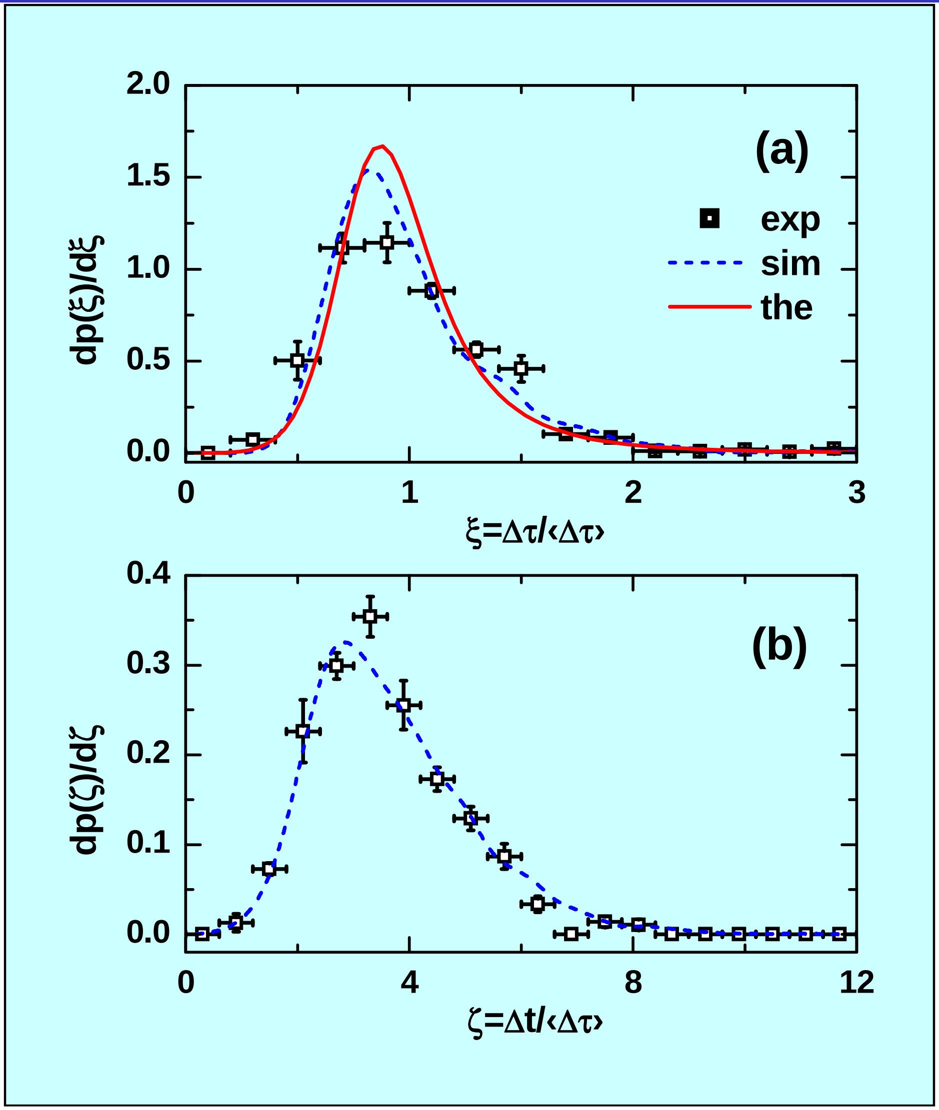
$$E_{sig}(t,\tau) \propto E(t)E(t-\tau).$$

And the measured signal on the spectrometer is

$$I_{FROG}(\omega,\tau) \propto \left| \int_{-\infty}^{\infty} E_{sig}(t,\tau)\exp(-i\omega t)dt \right|^2.$$



Temporal structure: spike width and spacing

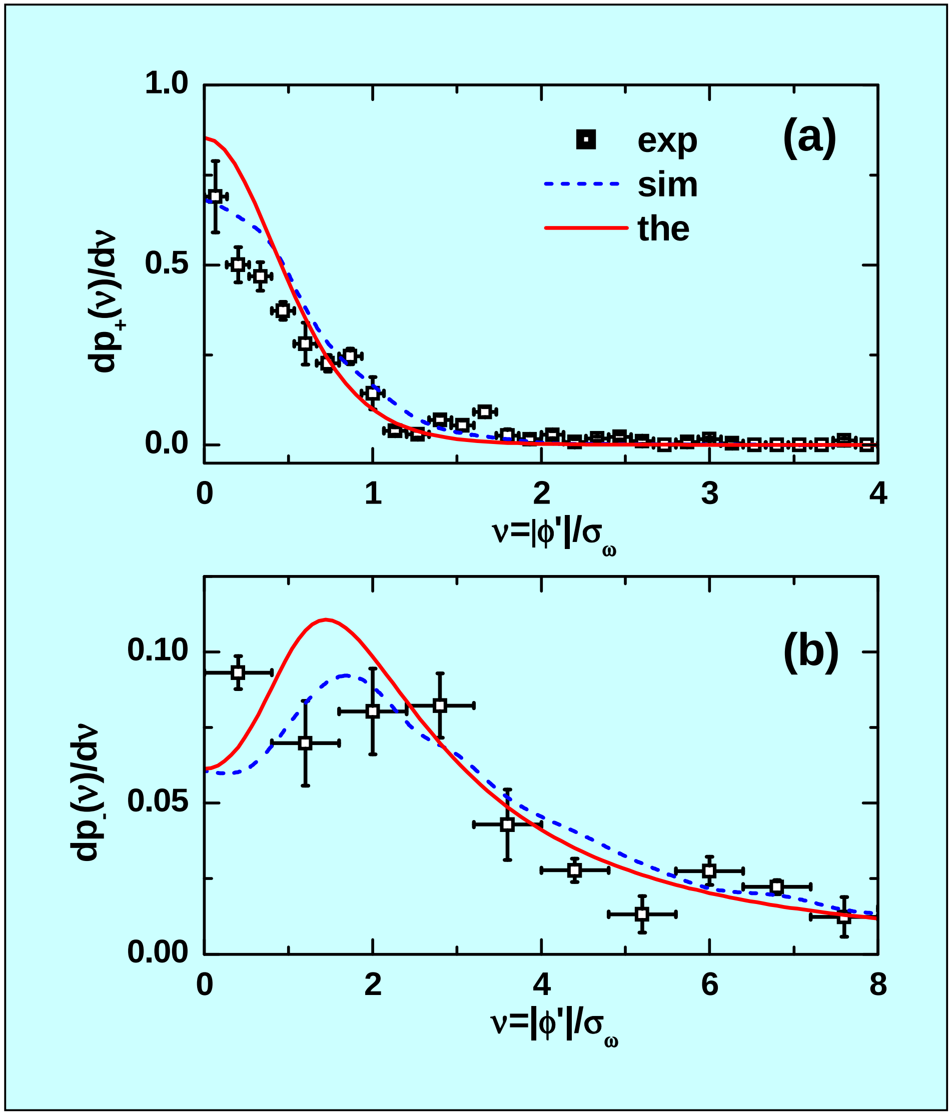


Follow statistics for chaotic light exactly.

$\langle \Delta \tau \rangle = 52$ fs

Li et al., PRL 91 243602 (2003).

Derivative of phase (frequency)



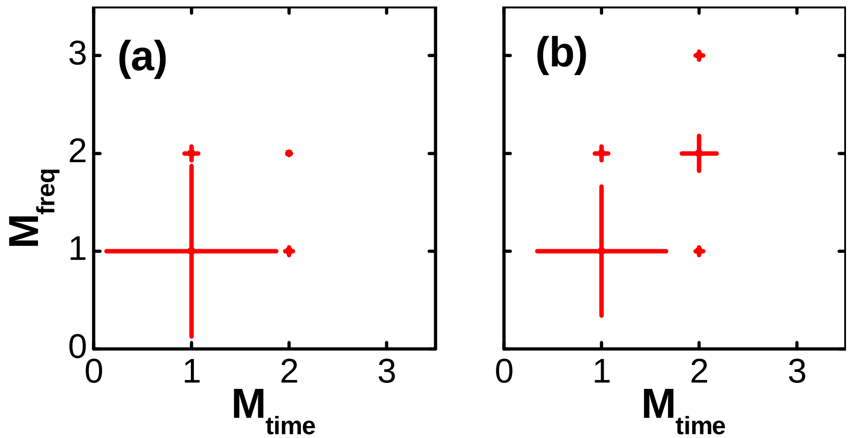
Each intensity spike is a coherence mode.

σ_ω=0.0094 rad/fs

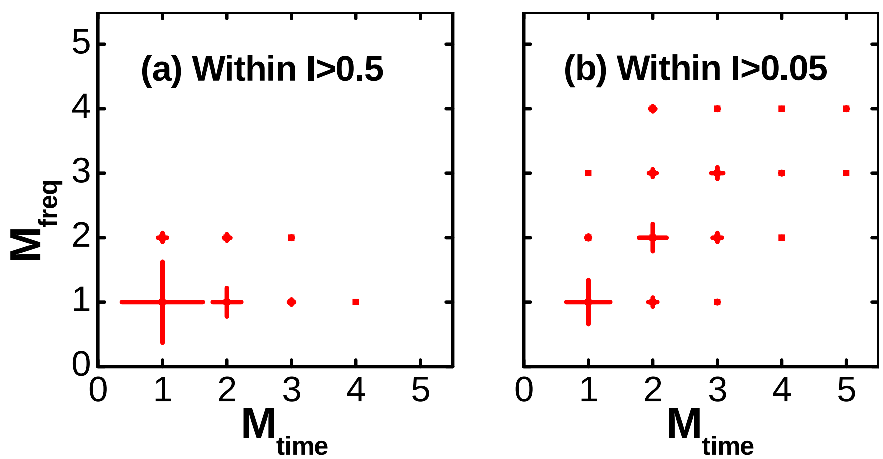
Li et al., PRL 91 243602 (2003).

Number of spikes in the two domains

Experiment B



Experiment A



Correlation between the time and frequency domains

		Measured	Calculated
Rms width	Time $\bar{\sigma}_t$ (fs)	83	
	Frequency $\bar{\sigma}_\omega$ (mrad/fs)	11	
Rms spike width	Time $\langle \delta t \rangle$ (fs)	52	$\langle \delta t \rangle = 1/\sqrt{2}\,\bar{\sigma}_\omega = 64$
	Frequency $\langle \delta\omega \rangle$ (mrad/fs)	7.9	$\langle \delta\omega \rangle = 1/\sqrt{2}\,\bar{\sigma}_t = 8.5$
Average spike spacing	Time $\langle \Delta t \rangle$ (fs)	208	$\langle \Delta t \rangle = \sqrt{2\pi}\,/\,\bar{\sigma}_\omega = 228$
	Frequency $\langle \Delta\omega \rangle$ (mrad/fs)	20	$\langle \Delta\omega \rangle = \sqrt{2\pi}\,/\,\bar{\sigma}_t = 30$
Coherence range	Time T_{coh} (fs)	$T_{coh} = \sqrt{2\pi}\,\langle \delta t \rangle = 130$	$T_{coh} = \sqrt{\pi}\,/\,\bar{\sigma}_\omega = 156$
	Frequency Ω_{coh} (mrad/fs)	$\Omega_{coh} = \sqrt{2\pi}\,\langle \delta\omega \rangle = 19$	$\Omega_{coh} = \sqrt{\pi}\,/\,\bar{\sigma}_t = 21$
Mode #	M	$M = 2\,\bar{\sigma}_\omega\,\bar{\sigma}_t = 1.8$	$M = 1/(\sigma_W/\langle W \rangle)^2 = 2.6$

The two domains should have about the same number of intensity spikes